Measurement :
Math & Science I
An introduction

Lecture 2

Math is the Language of Science.

- This is most obvious in **physics**, the study of the physical world, the relationships between matter and energy.
- The principles of physics can be expressed in ordinary words. However, math is needed to make precise and accurate predictions, and to test them. It is also far quicker to express and explain relationships using math than using words.
- To uncover new dynamics or to attain deeper understanding scientists observe, experiment with careful measurement and analyze results. Math makes this possible and easier than using words alone.

More than one measurement is used to answer a question. Information is derived with issues around these 3 things:

1. Some numbers may be much larger or smaller than others: **magnitude** differs. Scientific notation helps
2. They might be measured using instruments with different units (inches or centimeters): need for **conversion** to another unit
3. What else does a number tell you? How real is your measure : **significance**

A number is just a number or is it?

- Experimentation and observations used to try to answer questions must be thoughtfully recorded.

- The way data is written holds a lot of information about accuracy of instruments and methods. Every number has a portion that is an estimate.

All measures have some error in them.

We have been discussing the example of counting tomatoes in order to answer a question.

A **count** is simple because it is a measure with no error per item counted. You are counting whole healthy tomatoes. (What is deemed healthy may be where error comes in)

Instruments used to make other measures (about mass or size for example) would have some error built in and you would have to choose the correct instrument based on your level of **accuracy** needed.

What if instead of number of tomatoes you where looking to see if more fertilizer affected size or density of the tomatoes ?

- How would you measure the size of the tomato or its density?

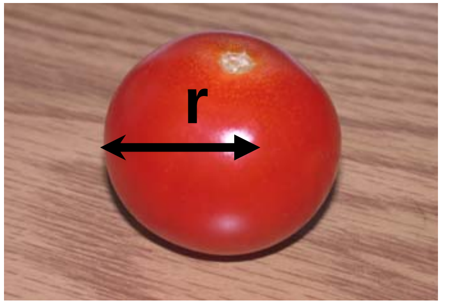
- **Displacing** a known **volume** of water is a good way to get the volume of an object that is challenging to measure. (they are not perfect spheres)

- **Density** is **mass/ volume** and if you can figure out the volume you then need to weigh the tomato to get its mass.

You could use formulas

- The formula for the volume of a sphere is

$$V = \frac{4\pi r^3}{3}$$



π is known information

r is the **radius** which is half the diameter of the sphere (half the width of our tomato) and is the only measured information you need.

A flexible measuring would give you the radius but how accurate would it be ?

Accuracy

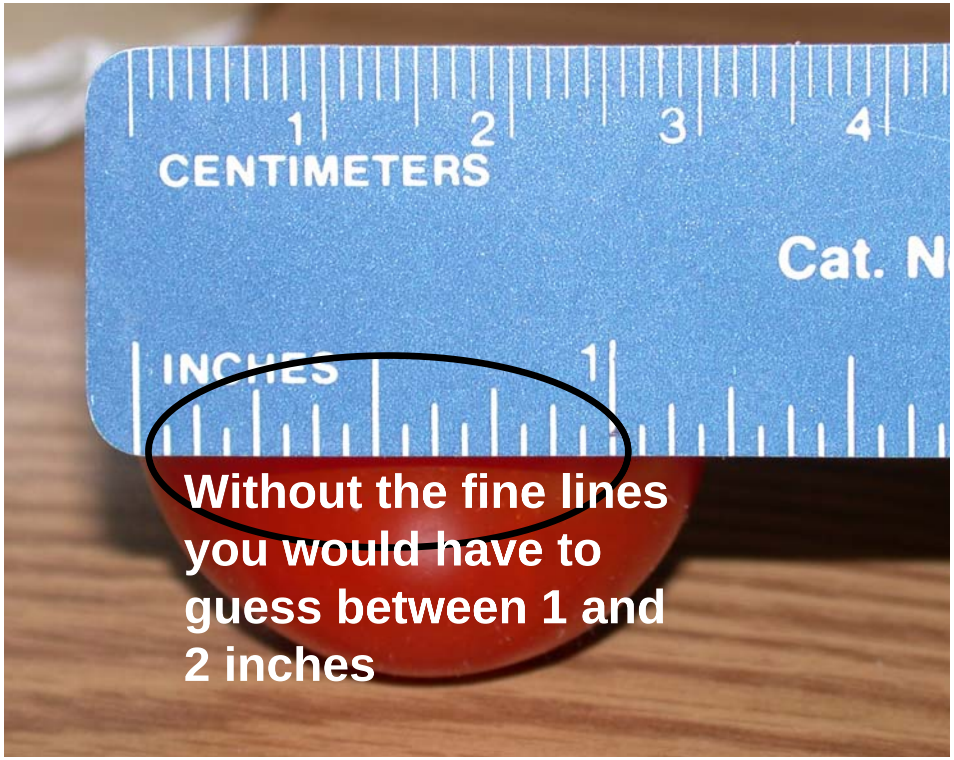
is the extent to which a <u>measured</u> value agrees with the <u>standard</u> value of a quantity.

This is limited by how close your measurement of a known standard is.

(The finer the gradations the smaller the error of estimate.)

Calibration tests and corrects for the level of inaccuracy.

- A ruler, tape measure or caliper with standard measures and fine gradations at the scale of interest would allow you to take an accurate measure.
- If you want the diameter of a small tomato you do not want to use a ruler with only inch or centimeter marks.



How ever you record information, it must be consistent and reveal where the error is.

- The tomato is 1inch and 3/16ths or

  (3/16 = 0.1875) 1.18750 inch

  More tomato measures might include 11/4 inch, or 1/5 so get rid of the fractions. (& this is where it is easy to see that the metric scale is easier to use)

  Why include the zero at the end?

It indicates the level of guess work. The number above has 6 significant digits in it. (all are significant)

Significance Rules

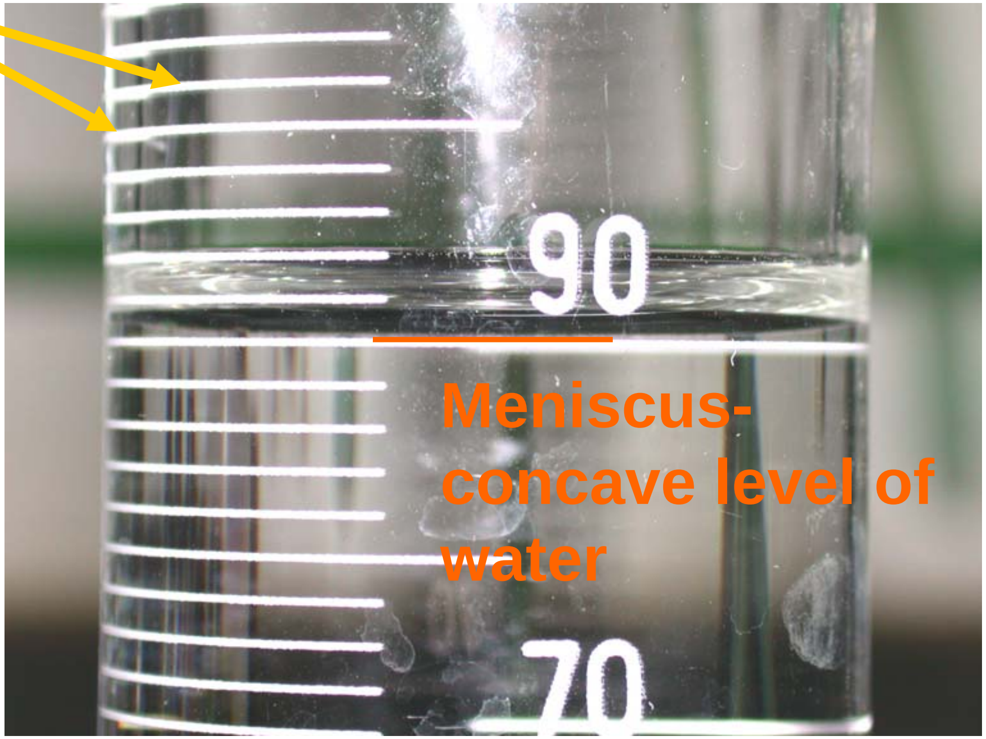
Valid digits are called significant digits.

They include the part of the number you are sure of and one estimated digit (the last).

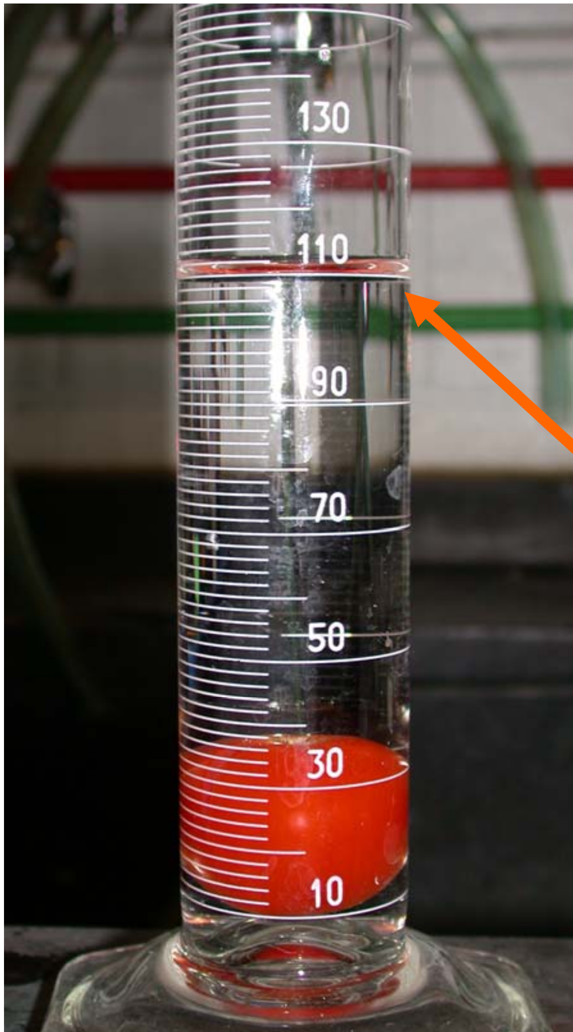
1) nonzero digits are always significant
2) All final zeros after the decimal point are significant
3) Zeros between two other significant digits are always significant
4) Zeros used solely for spacing the decimal point are not significant.

Get the volume by displacing water

- Here you are still limited by the measure marks on your instrument. In this case a graduated cylinder holding 90ml of water.
- Smallest gradation is 2ml.



Drop in the tomato and measure the displacement of water



The Meniscus is now at 110ml. It was at 90ml before adding the tomato which takes up 20ml (it displaces 20ml of water). This is best written as 20.0 cubic centimeters or 20.0cc .

Displacement is faster and doesn't involve too much handling of the tomatoes.

To get the **density** you must weigh them and divide the weight by the volume. You will then know the tissue per space.

(pithy tomatoes are probably less dense)

Which scale do you use?

How much does the average tomato weigh? Just like choosing a ruler limit guess work if possible. Choose the one that gives you the most measure at the level of variation.

Mass & Scale example:

Once you know the scale you want to use :

How accurate is the scale (also called a balance) I am about to use?

Measure the mass (weight) of standardized* weights to test how accurate the scale is. *(*weights of known mass appropriate to your scale)*

To repeatedly weigh them will tell the **precision** of the instrument and of your method.

Standard Weights:

10mg up to 100g

This standard weight is known to weigh 100g.

It appears to be accurate to one tenth of a gram.

Or, the weight might actually weigh 100.016g.

To know we will use the smaller standard weights of known milligrams.

How precise is the scale?

Precision

is the degree of exactness to which the measurement of a quantity can be reproduced.

How small is the error or estimated portion of your recording?

- This is limited by the smallest division of measure.
- This scale cannot be 100% accurate or precise in measures from 1 to 9 mg (which is 0.001-0.009g)
- The last digit here is an estimate and subject to error.
- With rulers and cylinders the last measure is accurate, the estimate is what is between them.

How accurate and how precise?

You would weight it more times to determine the level of error you would expect from this instrument.

The accuracy and precision level is to the hundredths place.

Each reading differs at the thousandths place, it has error.

This is an important concept, data must be truthful and real.

We can test this too, using a smaller standard weight. (50mg)

Weigh it a few times…

Again, this scale has a low level of accuracy or precision for items of less than 10 milligrams.

Statistics

- A statistic is a measure.
- We could calculate the average error of the scale used to make these mass measures.
- We would continue to repeat the weighing.
- We would calculate an average weight, then see how much each measure differed from the average weight.
- If they do not deviate much on average, then the mean represents the measure and the error is small.
- What if they deviate a lot? Error is large.

If the difference in mass between these two tomatoes represented the average you had among your experimental tomatoes. Then such a scale is ok to use. Why?

The difference is at the gram level. If the average difference among tomatoes was 0.002g would you continue to use this scale? *No*

When you record your data, you end the number to represent **accuracy.** Though calculations produce many digits you may not include them all when calculating.

Vantage point

- **Parallax** is the apparent shift in the position of an object when it is viewed from various angles.

- Many instruments require that you read them straight on at eye level. Parallax errors originate when read incorrectly from a different angle.

Measurements

- Most countries in the world and all scientists communicate by using the SI system, an adaptation of the metric system of measurement.
- It is easy, since all units are based on a power of 10.
- The standard unit of
  - **Time** is the second (s)
  - **Length** is the meter (m)
  - **Mass** is the gram (g)
- Length, mass and time are fundamental units that can be used to describe what we call derived units, combinations of fundamental units.

Example: speed is derived and is expressed in m/s.

Some Common SI Unit Prefixes

Prefix	Symbol	Fractions of the unit meter (m)	Examples using meters =1
pico	p	1/1,000,000,000,000 or 10^{-12}	picometer (pm)
nano	n	1/1,000,000,000 or 10^{-9}	nanometer (nm)
micro	μ	1/1,000,000 or 10^{-6}	micrometer (μm)
milli	m	1/1,000 or 10^{-3}	millimeter (mm)
centi	c	1/100 or 10^{-2}	centimeter (cm)
deci	d	1/10 or 10^{-1}	decimeter (dm)
		Multiples of the unit meter (m)	** Gram units*
tera	T	1,000,000,000,000 or 10^{12}	terameter (Tm)
giga	G	1,000,000,000 or 10^{9}	gigameter (Gm)
mega	M	1,000,000 or 10^{6}	megagram (Mg) *
kilo	k	1,000 or 10^{3}	kilometer (km)
hecto	h	100 or 10^{2}	hectometer (hm)
deka	da	10 or 10^{1}	Dekagram (dag) *

Scientific Notation & Magnitude

- This makes using very large and very small numbers easier.

- You can express any number as a number between 1 and 10, multiplied by a power of 10.

- A power of 10 is a small number in the upper right corner of the 10 called an exponent, it is the number of times ten is multiplied by itself and the number preceding it.

10^3 is 10 x 10 x 10 = 1,000

For example:

Using Scientific notation, 4000m can also be written so:

4.0×10^3 m

These two numbers have the same value, they are just written differently.

Four thousand meters is identical to
four multiplied by 10^3 m,
since 10^3 is the same as 1,000.

How to recognize the number:

To write in scientific notation:

- You move the decimal point, counting the number of places it moves, until only one nonzero number remains on the left of the decimal.
- (the number of counted places moved is the exponent of 10)

scientific notation form: $M \times 10^n$ where :

$1 \leq M < 10$ (M is greater than 1 and less than 10) and n is an integer (a whole number) equal to the decimal places counted.

Exponents: Powers of 10

- Depending on the **direction** the decimal is moved, 10 is raised to either a negative or a positive number. It is either 10 times smaller or larger.

One hundred meters: 100m is written 1.0×10^{2}m

One one hundredth of a meter,1/100m or 0.01m

is written 1.0×10^{-2}m

- A **negative** exponent indicates that the nonscientific notation form of the number had its decimal point moved to the right that many places. (It is a small number, a fraction of a unit, you are actually dividing by a power of 10)
- A **positive** exponent indicates that the decimal point was moved to the left to be written in with scientific notation. (This is a big number, a multiple of a unit)

Examples:

Change these numbers out of scientific notation and ask is this # big or small relative to M?

Look at the exponent's sign (+or-) for relative size.

	Size relative to M?		
5.25×10^{3} kilos	big	=	5,250 kilos
9.75×10^{-3} miles	small	=	0.00975 miles
8.01×10^{-3}m	small	=	0.00801 m
6.33×10^{2} m/s	big	=	633 m/s

How many significant digits are in each of these numbers?

Significant Digits

- The valid portions of any measurement are called the significant digits (or significant figures).
- The last digit is the one you are unsure of, it is the estimate.

For example:

If the smallest divisions on a meter stick are millimeters, you measure to the nearest millimeter. Then estimate any remaining length as a fraction of a millimeter.

An object that you measure to be 32 mm long is written as 32.0 mm. **The zero is important** it conveys the **estimated** and the **valid** digits.

Here are the rules

- Nonzero digits are always significant.
- All final zeros after the decimal point are significant.
- Zeros between two other significant digits are always significant.
- Zeros used solely for spacing the decimal point are not significant.
- **The result of any mathematical operation with measurements can never be more precise than the least precise measurement made.**
  - You first perform the math operations then round off to what corresponds with the least precise value involved.

Zeros

- To show where the decimal is, is not significant.
- If they follow a number larger than 0 they are usually significant.

<u>For example, look at these measurements:</u>

0.0026 kg has only 2 significant figures, 2 & 6.

0.002060 kg has four significant figures, 2060 the final zero is important to convey an estimate.

186,000 m : It is hard to know with this number. If all three zeros resulted from measure, there are 6 significant digits. But if 6 was the last digit recorded with accuracy, then there are only 3 significant digits in this measure. If you are not told the estimate of measure, then assume the last digit is significant.

Scientific Notation & Significance

If 186,000 m is written so:

1.86×10^5 m

it is understood that the last significant measure is 6 and has some error.

If written so:

1.86000×10^5 m

it is understood that the measure is real to the last zero, which has some error.

- It is up to the person making and recording the measurement to show which was the last digit to be measured as an estimate.
- This is another use of the scientific notation. All digits are significant that appear before the power of 10.
- For example from the previous slide in the first example, 1,8 and 6 are significant.
- In the second example the zeros are also significant, that is why they are included they reveal the accuracy of the measure.

Example

$1.86\underline{0}$ x10^{-5} m

$2.5\underline{0}$ x 10^{-5} m

+ $1.00\underline{0}$ x 10^{-5} m   Carry out the math and you get :

5.360 x 10^{-5} m

Properly write it, to show where the precision of measure stops and an estimate begins, you must match it with the least precise measure which is based on $2.5\underline{0}$ x 10^{-5} m the least precise.

You must round off to the hundredths place when you write the final calculation $5.3\underline{6}$ x 10^{-5} m

Without Scientific Notation?

5.36×10^{-5} m also equals 0.0000536 m

When written so, one can see that the four zeros are part of the number but really not part of the measure. They show magnitude.

Practice Examples

How many significant figures are there in each of these numbers?

- 4.056g
- 0.003g
- 3.030g
- 2.20g

Answers:

- **4.056**g 4: all 4 digits are significant
  - zeros between nonzero integers are significant
- 0.00**3**g 1: only the 3 is significant,
  - zeros show magnitude, where the decimal point is
- **3.030**g 4: all 4 digits are significant
  - zeros between nonzero integers and final zeros after the decimal point are significant
- **2.20**g 3: all three digits are significant,
  - Final zeros after the decimal point are significant

To derive the density of a tomato for our hypothetical experiment we would carryout the following:

Density = mass/ volume

Our 2 methods discussed create different numbers and levels of precision.

Using displacement of water

$$\frac{19.87\underline{5}\,g}{20.\underline{0}\ cc} = 0.\underline{9}9375\ g/cc$$

V = round up to 1.0g/cc

Using a ruler

1)divide by 2 for the radius (1/2 the diameter)
2)convert to cm,
3) calculate volume (V)
4) calculate density

1) D=1.18750inches   r= 1.18750in/2 = 0.59375in

2)Convert= 0.59375in x 2.54cm/in=1.508125cm

3) V = $\frac{4pr3}{3}$ = 14.360866cc = 14.360866cc

4) $\frac{19.875g}{14.360866cc}$ = 1.38387 g/cc   V=round up to 1.384g/cc

- How do the methods differ in outcome?
- How would you determine which one to use?
- How could you improve the methods?
- By improve I mean reduce error or increase precision, what would do that?