Measurement : Math & Science II

Examples of:

Scientific Notation

Conversions

Significance

Lecture 3

Helpful math website:

http://www.purplemath.com/modules/index.htm

Operations in Scientific Notation ($M \times 10^n$)

Addition and Subtraction: Exponents must be the same

- Units must be the same, convert any that need it. (for example if you have **mm** and **m** change them all to either mm or m)
- If exponents (^{n}) **are the same** (10^n) then you can add or subtract the values of M. 10^n remains the same.
- If the exponents **are not the same**, adjust the smaller before you can add or subtract the values of M. Do this by moving the decimal point of the smaller number until the exponents are equal. Adjust M accordingly.

Examples

Adding with like exponents

$5 \times 10^{8}m + 3 \times 10^{8}m =$ $8 \times 10^{8}m$

$4.2 \times 10^{-3}m + 1.2 \times 10^{-3}m =$ $5.4 \times 10^{-3}m$

Subtracting with like exponents

$4 \times 10^{9}m - 2 \times 10^{9}m =$ $2 \times 10^{9}m$

$4.2 \times 10^{-3}m - 1.2 \times 10^{-3}m =$ $2.0 \times 10^{-3}m$

Examples

<u>Adding with unlike exponents</u>

5×10^6m + 3×10^4m = change the smaller one

5×10^6m + 0.03×10^6m = 5.03×10^6m

you can double check it:

5,000,000 + 30,000 = 5,030,000 = 5.03×10^6m

<u>Subtracting with unlike exponents</u>

4×10^9m - 2×10^7m = change the smaller one

4×10^9m -0.02×10^9m = 3.98×10^9m

Operations in Scientific Notation
(M x 10 n)

Multiplication or Division: Exponents need not be the same

- Units must be the same or able to be merged, convert any that need it and change the units.

Multiplication

- **Multiply** the values of M and **then add** exponents together. Then you can add or subtract the values of M.

Division

- **Divide** the values of M and then **subtract** the exponent from the divisor (bottom of fraction; what you are dividing by) from the exponent of the dividend (top of fraction; what is being divided).

Examples

Multiplying

$(4 \times 10^2 m) \times (2 \times 10^3 m) = 8 \times 10^5 m$

This is the same as this:

$400m \times 2{,}000m = 800{,}000m = 8 \times 10^5 m$

Dividing

$$\frac{(6 \times 10^4 m)}{(2 \times 10^2 m)} = 3 \times 10^2 m$$

This is the same as $\frac{60{,}000m}{200m} = 300m = 3 \times 10^2 m$

Rules of Signs & Absolute Values of Numbers

Addition

1) When numbers have the same sign, sum their absolute value and write the common sign.
2) When numbers have different signs, subtract their absolute values and use the sign of the number with the larger absolute value.

Subtraction

* When you subtract a negative number you are adding the absolute value of it.

Adding

- + - = -   (-1) + (-1)=(-2)

+ + +=+   (+2) + (+1) =(+3)

+ + - = sign of the greater number after subtraction

(+5) + (-10) = -5

(-2) + (+3) = +1

Subtracting

+ - - = +   (+4) – (-1) = (+4) + 1= 5

- - - = sign of the greater number after subtraction

(-3) – (-2) = (-3) + 2= (-1)

+ - + = sign of the greater number after subtraction

(+5) – (+ 7) = (-2)

Conversions

Units

When you record data, the number is followed with a **unit** label describing what type of measurement you are making.

In solving problems, you'll find that the numbers involved often have different units.

You then use conversion factors to do your work.

- For **Length** you might use units like the centimeter (cm), inch (in), foot (ft) or kilometer (k) It is linear through space, from point A to point B. But can be squared or cubed.

- For **Volume** you might use units like the milliliter (ml), cubic centimeter(cc) or cubic foot (ft^3) It is cubic space like an x, y, z graph (3 dimensional)

- For **speed** you might use seconds (s), hour (hr) or nanoseconds (ns)

  It is also linear but through time, and can be raised to a power.

Help

- Tables, charts or conversion equations exist to help you express a measurement differently.

- <u>You are not expected to know these things off the top of your head.</u>

  - As you work with conversions, you will just start to know some factors.

  - In some courses you are asked to learn them. In here you might have to learn a few: you will be given that information.

Most of the time you are multiplying the *number you wish to change by a fraction equal to 1.

The units you want to be rid of are on the bottom and the ones you want to end up with are on the top of the fraction.

For example a familiar thing:

If 1 cup has 8 tablespoons (tbs) in it, **how many tablespoons are in ¼ cup?**

$$\frac{1}{4}\ \cancel{\text{cup}} \times \frac{8\ \text{tbs}}{\cancel{\text{cup}}} = \frac{8\ \text{tbs}}{4} = 2\ \text{tbs}$$

this fraction = 1

The conversion factor turn cups into tablespoons and it is 8 tablespoons per cup. (the unit cup cancels out when you multiply the fraction by the number expressed in cups, because you divide by cups)

A Different Example

You need to determine the average temperature of a summer day in Boston, but the data recorded was sometimes taken in Fahrenheit and sometimes in Celsius, you would choose one (C or F) to convert all numbers into before calculating the average.

You are not changing the value of the data, you are only changing how it is represented.

The conversion equation to go from C to F is

$9/5\ (C) + 32 = F$

It is almost 2x the degrees Celsius plus 32, it is a good way to estimate a check of your calculations.

It is a reminder that when you carry out mathematics you should try to imagine the magnitude of where the calculation is going.

Temperature Readings

Cannot add mixed units!

Use the equation: **9/5(C) + 32 = F**

92F		92F
32.2C →	9/5(32.2) + 32 =	89.96 F
90F		90F
89F		89F
33C →	9/5(33) + 32 =	91.4 F
31C →	9/5(31) + 32 =	87.8 F
93F		93F
+ 30C →	9/5 (30) + 32 =	+ 86 F
?		719.16 **F**/8=

Average :89.895 F

Round up to 90 F

Temperature Scales

Kelvin (K)
1K = 1ºC but the 0 point differs by 273.15º 0K= -273.15ºC
Absolute zero= 0 Water is ice at 273.15K, water boils at 373.15K

Celsius (C) is the common metric scale,
Absolute zero= -273.15 C Water is ice at 0C, water boils at 100ºC
Water's freezing point: 0 Celsius = 273.15K, 0 Celsius= 32 F

Rankine (ºR) 1 ºR = 1 ºF but the 0 point differs by 459.67º

Fahrenheit (F) Water freezes at 32ºF, water boils at 212º F

Metric is easy

- If you have mixed units, but they are all in the metric system you will have an easier conversion calculation.
- The metric system uses 10 fold changes to shift from one scale of measure to a larger or smaller one.
- This makes the math cleaner, and even portions of words which indicate magnitude are repeated. As seen in the next slide.

For example: the prefix **Milli** represents 1/1000 of something, a milliliter is 1/1000 of a liter, a milligram 1/1000 of a gram, a millisecond 1/1000 of a second.

Some Common SI Unit Prefixes

Prefix	**Symbol**	**Fractions of the unit meter**	**Examples using meters =1**
pico	p	1/1,000,000,000,000 or 10^{-12}	picometer (pm)
nano	n	1/1,000,000,000 or 10^{-9}	nanometer (nm)
micro	μ	1/1,000,000 or 10^{-6}	micrometer (μm)
milli	m	1/1,000 or 10^{-3}	millimeter (mm)
centi	c	1/100 or 10^{-2}	centimeter (cm)
deci	d	1/10 or 10^{-1}	decimeter (dm)
		Multiples of the unit meter	* *Gram examples*
tera	T	1,000,000,000,000 or 10^{12}	terameter (Tm)
giga	G	1,000,000,000 or 10^{9}	gigameter (Gm)
mega	M	1,000,000 or 10^{6}	megagram (Mg) *
kilo	k	1,000 or 10^{3}	kilometer (km)
hecto	h	100 or 10^{2}	hectometer (hm)
deka	da	10 or 10^{1}	Dekagram (dag) *

Example problem

Conversion between units:

What is the equivalent of 1200 millimeters in meters?

Solution from the previous slide you know that the conversion factor is

1 millimeter = 1×10^{-3} meter, therefore:

$$(1200\ mm)\ \frac{(1 \times 10^{-3} m)}{1 mm} = 1200 \times 10^{-3} m$$

Move the decimal point to the left until you have a number between 1 and less than 10.

you are left with 1.2×10^{0}m = 1.2 m

There is another way to do this, or to check yourself:

Go with what you know…

<u>1 millimeter = 1 x 10 $^{-3}$ meter, therefore:</u>

The negative exponent indicates that you are actually multiplying by a fraction of a unit. A millimeter is a fraction of a meter, exactly 10 $^{-3}$ meter, or 1/10x10x10 m= 1/1000 m

$$(1200\ \cancel{mm}) \times \frac{\mathbf{1\ m}}{\mathbf{1{,}000\cancel{mm}}} = 1200m/1{,}000 = 1.2m$$

If you know there are 1,000 mm in a meter, divide your number in mm by 1,000mm to convert to meters. (just remember to keep the m unit afterwards)

The Reverse

Convert from meters to millimeters

How many mm is 0.5m ?

The conversion factor is 1 x 10^3 mm/m

Or 1,000mm/m (←*again your fraction equal to 1*)

Working Solution :

0.5~~m~~ x (1 x 10^3 mm/~~m~~) = 0.5 x 10^3 mm

Now move the decimal point to the right once to write this properly with scientific notation.

Final answer: 5.0 x 10^2mm

Or, eliminate the notation and it write it as **500mm**. (which is half a meter)

Check yourself with what you know, ½ meter is 500mm.

Adding and subtracting with unlike units:

2 inches + 5cm =

First you convert inches to cm or the reverse.

Conversion is 1inch = 2.54cm

$$2 \text{ inches} \left(\frac{2.54\text{cm}}{\text{inch}}\right) = 5.08\text{cm}$$

(inches cancelled out)

Now they can be added together:

5.08cm + 5cm = 10.08cm

round off to 10cm

Where to stop writing digits?

Don't forget significance

- Calculators can generate long numbers.
- How do you know how to properly record a number ?
- Limitations to writing numbers originates with the instruments used to measure and record.
- Always go with the least significant digit.